Mon. Not. R. Astron. Soc. **000**, 1–9 (2006) Printed 5 February 2008 (MN LaTeX style file v2.2)

Six Myths on the Virial Theorem for Interstellar Clouds

Javier Ballesteros-Paredes*
Centro de Radioastronomía y Astrofísica, UNAM. Apdo. Postal 72-3 (Xangari), Morelia, Michocán 58089, México

Submitted to MNRAS, 5 February 2008

ABSTRACT

The interstellar medium is highly dynamic and turbulent. However, little or no attention has been paid in the literature to the implications that this fact has on the validity of at least six common assumptions on the Virial Theorem (VT), which are: (i) the only role of turbulent motions within a cloud is to provide support against collapse, (ii) the surface terms are negligible compared to the volumetric ones, (iii) the gravitational term is a binding source for the clouds since it can be approximated by the gravitational energy, (iv) the sign of the second-time derivative of the moment of inertia determines whether the cloud is contracting ($\ddot{I} < 0$) or expanding ($\ddot{I} > 0$), (v) interstellar clouds are in Virial Equilibrium (VE), and (vi) Larson's (1981) relations (mean density-size and velocity dispersion-size) are the observational proof that clouds are in VE. Turbulent, supersonic interstellar clouds cannot fulfill these assumptions, however, because turbulent fragmentation will induce flux of mass, moment and energy between the clouds and their environment, and will favor local collapse while may disrupt the clouds within a dynamical timescale. It is argued that, although the observational and numerical evidence suggests that interstellar clouds are not in VE, the so-called "Virial Mass" estimations, which actually should be called "energy-equipartition mass" estimations, are good order-of magnitude estimations of the actual mass of the clouds just because observational surveys will tend to detect interstellar clouds appearing to be close to energy equipartition. Similarly, order of magnitude estimations of the energy content of the clouds are reasonable. However, since clouds are actually out of VE, as suggested by asymmetrical line profiles, they should be transient entities. These results are compatible with observationally-based estimations for rapid star formation, and call into question the models for the star formation efficiency based on clouds being in VE.

Key words: ISM: general – clouds – kinematics and dynamics – turbulence – stars: formation

1 INTRODUCTION

Interstellar clouds are thought to be turbulent and supersonic. Their Mach numbers range from a 1 ($T \sim 7000$–8000 K, H I clouds) through 10 ($T \sim 10$ K, molecular clouds forming low-mass stars) to 50 ($T \sim 10$–50 K, molecular clouds forming high-mass stars). Since supersonic turbulent motions carry mass and produce large-amplitude density fluctuations, turbulent fragmentation[1] is expected to occur in the interstellar medium.

A useful tool for describing the overall structure of interstellar clouds is the scalar Virial Theorem (VT). It is obtained by dotting the momentum equation by the position vector and integrating over the volume of interest (§2).

[1] Turbulent fragmentation is defined as the process through which a chaotic velocity field produces a clumpy density structure in the gas within a few dynamical timescales (see e.g., von Weizsäcker 1951; Sasao 1973; Scalo 1988; Padoan 1995; Ballesteros-Paredes et al. 1999a; Klessen et al. 2000; Heitsch et al. 2001; Ballesteros-Paredes 2004b).

* E-mail: javier@astrosmo.unam.mx

Being directly derived from the momentum equation, the VT always holds for any parcel of fluid.

Virial Equilibrium (VE) is a restrictive condition of the VT, and it is defined by the condition that the parcel of fluid under study has a second time derivative of the moment of inertia equal to zero. It has been invoked extensively to analyze the stability of interstellar clouds. However, since in a trans- or super-sonic turbulent interstellar medium, clouds should be redistributing their mass as a consequence of their own turbulent motions, it already seems difficult to achieve VE in a supersonic, turbulent cloud.

Other simplifications, such as the assumption that the cloud is isolated, or that the surface terms in the VT are negligible, are frequently made in many (if not in most) astrophysical studies of the VT. Those simplifications have been thought to be applicable to molecular clouds and their substructure for nearly three decades (e.g., the textbooks by Spitzer 1978; Shu 1991; Stahler & Palla 2005; Lequeux 2005, and references therein), maybe as a consequence of the old idea that in the ISM, "all forces are in balance and the medium is motionless, with no net acceleration" (Spitzer 1978, Chap. 11), in which "the observational evidence" seemed to be consistent with the expectation that interstellar "clouds tend toward pressure equilibrium" (Spitzer 1978).

The possible inapplicability of these assumptions has been mentioned in passing in some previous papers (e.g., Ballesteros-Paredes et al. 1999a; Shadmehri et al. 2002; Ballesteros-Paredes 2004a) but no attention has been paid in general to its implications. Thus, in the present paper I discuss in detail the applicability of those assumptions for molecular clouds and their cores. In §2 I write explicitly the VT for fluids in its Lagrangian and Eulerian forms. In §3 I discuss the six more common assumptions of the VT and their validity in a turbulent environment. In §4 I explain why even though clouds are not in VE, they appear to be in energy equipartition, and argue that asymmetries in the line profiles are the evidence for clouds out of VE. Finally, in §5 I draw the main conclusions.

2 THE VIRIAL THEOREM

The Virial Theorem can be derived from the momentum equation, by dotting it by the position vector $\mathbf{x}$ and integrating it over the volume of interest. Althoughit is usually written in its Lagrangian form, i.e., by following the mass (see, e.g., Spitzer 1978; Shu 1991; Hartmann 1998), it can be also obtained in its Eulerian form, i.e., by fixing the volume in space (see, e.g., Parker 1979; McKee and Zweibel 1992), obtaining

$$\frac{1}{2}\frac{d^2 I_E}{dt^2} = 2\bigg(E_{\rm kin}+E_{\rm int}\bigg)-2\bigg(\tau_{\rm kin}+\tau_{\rm int}\bigg)+M+\tau_M-W-\frac{1}{2}\frac{d\Phi}{dt} \quad (1)$$

$$\frac{1}{2}\frac{d^2 I_L}{dt^2} = 2\bigg(E_{\rm kin}+E_{\rm int}\bigg) - 2\tau_{\rm int} + M + \tau_M - W \quad (2)$$

where $I = \int_V \rho r^2 dV$ is the moment of inertia of the cloud (subindexes E and L in eqs. [1] and [2] stand for Eulerian and Lagrangian, respectively), $E_{\rm kin} = 1/2 \int_V \rho u^2 dV$ and $\tau_{\rm kin} = -1/2 \oint_S x_i \rho u_i u_j \hat{n}_j dS$ are the kinetic energy of the cloud and the kinetic stresses evaluated at the surface of the cloud, respectively, $E_{\rm int} = 3/2 \int_V P dV$ is the internal energy, $\tau_{\rm int} = -1/2 \oint_S x_i P \hat{n}_i dS$ is the pressure surface term, $M = 1/8\pi \int_V B^2 dV$ is the magnetic energy, $\tau_M = 1/4\pi \oint x_i B_i B_j \hat{n}_j dS$ is the magnetic stress at the surface of the cloud, $W = \int_V \ x_i \rho \ \partial\phi/\partial x_i \ dV$ is the gravitational term, with ϕ being the gravitational potential, and $\Phi = \oint_S \rho u_i r^2 \hat{n}_i dS$ is the flux of moment of inertia through the surface of the cloud. In the previous equations, ρ, u_i, B_i, P, and $\hat{n}$ are the density, the i^{th} component of the velocity u, the i^{th} component of the magnetic field B, the pressure, and a unitary vector perpendicular to the surface S that surrounds the volume V, over which the integrals are performed, respectively. In the notation above it is used the Einstein convention, where repeated indexes are summed.

3 THE COMMON ASSUMPTIONS

Various assumptions for the terms involved in eqs. (1) and (2) have been made in the literature. Some of them, indeed, have been converted into myths, since their applicability to interstellar clouds not only is not demonstrated, but it is not even questioned, either in textbooks, or research papers.

3.1 First assumption: The kinetic energy is generally a term of support

It is almost universally considered in the literature that the turbulent (or kinetic) energy, $E_{\rm kin} = 1/2 \int \rho u^2 \ dV$, provides support to clouds against collapse. While this is true for a system of particles, and partially valid if the kinetic energy is in the form of large-scale expansion and/or rotation, it is by no means certain that all the kinetic energy available will help against collapse in a system where turbulent fragmentation can occur, as discussed below.

This idea has its origins in Chandrasekhar (1951), who proposed that in the analysis of the gravitational instability the turbulent velocity field should be included. In his description, an effective sound speed is introduced, given by

$$c_{\rm eff}^2 = c_s^2 + \frac{1}{3} u_{\rm rms}^2 \quad (3)$$

where c_s is the sound speed, and $u_{\rm rms}$ is the velocity dispersion of the turbulent motions (see, e.g., Klessen et al. 2000; Mac Low and Klessen 2004, for a review). This description is valid only if (a) turbulent motions are confined to scales much smaller than the size of the cloud (Ballesteros-Paredes et al. 1999a), and (b) such motions do not produce new, smaller-scale Jeans-unstable density enhancements. The first hypothesis disregards one of the main features of turbulent flows in general (e.g., Kolmogorov 1941; Lesieur 1990), and of interstellar clouds in particular (Larson 1981), namely, that the largest velocities occur at the largest scales. An attempt to include this fact has been proposed by Bonazzola et al. (1987), who suggested including the value of the rms velocity dispersion *at each scale* $l \propto 1/k$, i.e.,

$$c_{\rm eff}^2(k) = c_s^2 + \frac{1}{3}\langle u(k)\rangle^2 \tag{4}$$

where k the wavenumber corresponding to the scale l, and $\langle u(k)\rangle$ is given by the energy spectrum $E(k) = Ck^{-\delta}$ as

$$\langle u(k)\rangle^2 = \int_k^\infty E(k)dk = \frac{C}{1-\delta}k^{1-\delta} \ . \tag{5}$$

where C is a constant and δ is the spectral index. The second condition, i.e., that turbulent motions do not produce Jeans-unstable density enhancements, has the underlying complication that motions at scales larger than $l \sim 1/k$ will be very anisotropic with respect to structures of size l. Those modes will produce shear (through vortical modes) or compressions (through compressible modes) to the structures[2]. Compressions in particular reduce the local Jeans mass (Sasao 1973; Hunter & Fleck 1982), and can induce local collapse. Thus, a fraction of the turbulent kinetic energy is involved in promoting collapse, rather than opposing it.

By decomposing the velocity field in its solenoidal and compressible components, the kinetic energy modes that provide support to the clouds are those having divergence larger or equal to zero,

$$\nabla \cdot u \geqslant 0. \tag{6}$$

This includes the solenoidal modes ($\nabla \cdot u = 0$), and the expansional component of the compressible modes ($\nabla \cdot u > 0$). In other words, the precise result of collapse or support must then reflect the balance between all the agents that favor collapse against those agents that provide support. In the first group, not only the gravitational energy should be included, but also the kinetic energy involved in the compressible modes ($E_{{\rm kin},\nabla\cdot u<0}$), versus the kinetic energy involved in the expansional and rotational modes ($E_{{\rm kin},\nabla\cdot u\geqslant 0}$).

3.2 Second assumption: The surface terms are negligible

It is frequently found in the literature that the surface terms are neglected altogether, especially in observational work (Larson 1981; Myers & Goodman 1988; Fuller & Myers 1992), mainly because there is not a direct way of measuring them observationally, although it is also a common practice in theoretical studies (e.g., Chandrasekhar & Fermi 1953; Parker 1969) and textbooks (e.g., Spitzer 1978; Parker 1979; Shu 1991; Stahler & Palla 2005). This assumption is based on the idea that self-gravitating clouds may be considered isolated because then their internal energies dominate the dynamics. The most notable exception is the thermal pressure surface term $\tau_{\rm int}$, which is frequently invoked for "pressure confinement" (e.g., McCrea 1957; Keto & Myers 1986; Bertoldi & McKee 1992; Yonekura et al. 1997). Although some works have considered, by analogy, the possibility of turbulent pressure confinement by means of the term $\tau_{\rm kin}$ (e.g., McKee and Zweibel 1992), such confinement of a cloud is difficult to achieve because the large-scale turbulent motions are anisotropic and will in general distort or even disrupt the cloud (Ballesteros-Paredes et al. 1999a).

Although we cannot measure the surface terms from observations, the possibility that they are as important as their corresponding volumetric terms suggest to investigate into numerical simulations of the interstellar medium. In fact, Ballesteros-Paredes & Vázquez-Semadeni (1997) found that for an ensemble of clouds in two-dimensional simulations of the interstellar medium at a kiloparsec scale, the surface terms have magnitudes as large as those of the volumetric ones (for three-dimensional simulations, see also Shadmehri et al. 2002; Tilley & Pudritz 2004; Dib et al. 2006). This result suggests that, on one hand, either surface and volumetric terms are of comparable importance in shaping and supporting the clouds. On the other hand, it suggests that clouds must be interchanging mass, momentum and energy with the surrounding medium. In such an environment, the meaning of thermal or ram pressure confinement is not clear, since motions at all scales must morph and deform the cloud.

3.3 Third assumption: The gravitational term is the gravitational energy

The gravitational term entering the VT is written as

$$W = -\int_V x_i\ \rho\ \frac{\partial\phi}{\partial x_i}\ dV. \tag{7}$$

Splitting-up the gravitational potential as the contribution from the cloud itself ($\phi_{\rm cloud}$), plus the contribution from the outside ($\phi_{\rm ext}$),

$$\phi = \phi_{\rm cloud} + \phi_{\rm ext}\ , \tag{8}$$

the gravitational term can be written as:

$$W = E_g - \int_V x_i\ \rho\ \frac{\partial\phi_{\rm ext}}{\partial x_i}\ dV\ , \tag{9}$$

where $E_g = -1/2\int_V \rho\phi\ dV$ is the gravitational energy of the cloud alone, since the volume of integration of $\phi_{\rm cloud}$ and the volume V of the integral coincide (e.g., Chandrasekhar & Fermi 1953). The second term in the right-hand side is usually either implicitly or explicitly assumed to be negligible compared to the first term, and the gravitational term W is then assumed to equal the gravitational energy E_g. This is valid only if the distribution of mass is spheroidal, or if the medium outside the cloud is tenuous such that its contribution to the potential is negligible. However, clouds are more similar to irregular fractals with arbitrary shapes (frequently filamentary) than to spheroids (e.g., Falgarone et al. 1991), and the contribution from the external gravitational field may not be negligible, giving rise to tidal torques. Although up to now there is no observational estimation of the contribution of the external mass to the gravitational term for any interstellar cloud, mass estimates for for H I "envelopes" around molecular clouds are of the same order of magnitude than the mass of the molecular clouds themselves (e.g., Williams & Maddalena 1996;

[2] It should be remembered that compressible and vortical modes are coupled, and they exchange energy (e.g., Sasao 1973; Vázquez-Semadeni et al. 1996; Kornreich & Scalo 2000; Elmegreen & Scalo 2004).

Moriarty-Schieven et al. 1997). Thus, it is not difficult to realize that the contribution of the second term in the right-hand side of eq. (9) to the gravitational term W can be of the same order of magnitude as the gravitational energy E_g.

Similar arguments can be made for the interiors of molecular clouds: even though we can approximate their shapes as (triaxial) spheroids (Jijina et al. 1999), embedded molecular cloud cores are subject not only to their own self-gravity, but also to the tidal forces from their parental molecular cloud. Thus, it is not clear that the tidal forces represented in the second term of eq. (9) will be negligible, and the assumption that the gravitational term equals the gravitational energy of the cloud seems unjustified.

What is the meaning and the effect of the second term of eq. (9) on the energy budget of the clouds? It can be seen that it involves the gradient of the external potential. For non-symmetrical distributions of mass, this term is out of balance even if the distribution of mass inside the cloud is symmetric. Thus, this term represents the tidal forces over the mass contained in the volume V, and it can be split up into three terms:

$$\int_V x_i\ \rho\ \partial\phi_{\rm ext}/\partial x_i\ dV = \int_V \partial(\phi_{\rm ext}\ x_i\ \rho)/\partial x_i\ dV - 3\int \phi_{\rm ext}\ \rho\ dV - \int \phi_{\rm ext}\ x_i\ \partial\rho/\partial x_i\ dV\ . \qquad (10)$$

Using the Gauss theorem, the first term on the right-hand side of eq. (10) can be interpreted as the gravitational pressure evaluated at the boundary of the cloud,

$$\int_V \frac{\partial(\phi_{\rm ext}\ \rho\ x_i)}{\partial x_i}\ dV = \oint_S (\phi_{\rm ext}\ \rho\ x_i)\ \hat{n_i}\ dS. \qquad (11)$$

The second term, by similarity with the gravitational energy, can be interpreted as three times the work done to assemble the density distribution of the cloud against the external mass,

$$E_{g,\rm ext} = 3\int \phi_{\rm ext}\ \rho\ dV. \qquad (12)$$

Finally, the last term in eq (7),

$$\int \phi_{\rm ext}\ x_i\ \frac{\partial\rho}{\partial x_i}\ dV \qquad (13)$$

involves the gradient of the density field inside the cloud. Although there is no clear interpretation of this term, it is worthwhile noting that its contribution is null for a homogeneous distribution of mass inside the volume V of integration.

From the numerical point of view, several studies (Ballesteros-Paredes & Vázquez-Semadeni 1997; Ballesteros Paredes 1999; Shadmehri et al. 2002; Tilley & Pudritz 2004; Dib et al. 2006) have found that the gravitational term can be negative or positive. If negative, it will be a confining agent. If positive, its overall action will be to contribute to the disruption of the cloud/core.

3.4 Fourth assumption: the sign of $\ddot{I}$ defines whether the cloud is collapsing or expanding

It is frequently argued in the literature that if $\ddot{I} > 0$, the cloud must be expanding, while $\ddot{I} < 0$ implies that the cloud is contracting. Equilibrium is assumed to occur when $\ddot{I} = 0$.

This idea rests on the fact that the gravitational energy E_g has a negative sign, and it is a confining agent, while the internal energies (thermal, kinetic, or magnetic) are positive, and they are assumed to act as supporting agents against collapse. By neglecting the other terms, it can be assumed that if the gravitational energy is larger than the sum of the internal energies, the sign of the right-hand side of eq. (2) is negative. Physically, if gravity wins, the cloud collapses.

Although energetically this is true, it is not hard to find an example in which an expanding cloud has a negative second time derivative of the moment of inertia. Assume a sphere with constant density and fixed mass M. Its moment of inertia is

$$I = \frac{3}{5}\pi M R^2. \qquad (14)$$

If its size varies with time, for instance, as a power law,

$$R = R(t) = R_0\left(\frac{t}{t_0}\right)^{\gamma}, \qquad (15)$$

its second time derivative is given by

$$\ddot{I} = \frac{6M}{5}\left(\frac{R_0}{t_0}\right)^2\left(\frac{t}{t_0}\right)^{2\gamma-2}\gamma\left(2\,\gamma - 1\right), \qquad (16)$$

which is negative if $0 < \gamma < 1/2$, even though it is expanding. In general, $\ddot{I}$ has been treated in the literature as if it were $\dot{I}$.

3.5 Fifth assumption: interstellar clouds are in Virial Equilibrium

The definition of Virial Equilibrium is that the left-hand side of the Lagrangian Virial Theorem (eq. [2]) equals zero:

$$\ddot{I}_L = 0 \qquad (17)$$

(see, e.g., Spitzer 1978). Although there are some observational papers showing molecular clouds out of VE (e.g., Carr 1987; Loren 1989; Heyer et al. 2001), it is frequently encountered in the literature the VE assumption for MCs and their substructure (e.g., Myers & Goodman 1988; McKee 1999; Krumholz & McKee 2005; Ward-Thompson et al. 2006; Tan et al. 2006). As mentioned in the introduction, this statement is based on the (old) idea that all the forces (in the ISM) should be in balance, and the medium should have no net acceleration (see e.g., Spitzer 1978, , Chap. 11). However, both observational (Jenkins et al. 1983; Bowyer et al. 1995; Jenkins & Tripp 2001; Jenkins 2002; Redfield & Linsky 2004), and numerical (Vázquez-Semadeni et al 2003; Mac Low et al. 2005; Gazol, Vázquez-Semadeni & Kim 2005) studies have found that the ISM is not in strict pressure balance, but exhibits strong pressure fluctuations, and in fact the turbulent ram

pressure is significantly larger than the thermal one (e.g., Boulares & Cox 1990).

From an observational point of view, it cannot be demonstrated that clouds are in VE because neither the detailed three-dimensional density structure of molecular clouds, nor the time derivatives for interstellar clouds can be measured observationally. From a theoretical point of view, the VE assumption has strong implications: either supersonically turbulent clouds are not redistributing their mass inside, or the way in which the time-derivative of the moment of inertia ($\dot{I}$) vary (which can be interpreted as the time-variation of their mass redistribution) is constant. Both statements seem implausible in a highly dynamical, non-linear ISM. McKee and Zweibel (1992), for instance, have recognized the difficulty of achieving VE in a turbulent medium, because turbulent motions carry fluid elements, redistributing their mass.

From the numerical point of view, fortunately, it is possible to calculate all the terms of the VT for clouds, since we know all the variables involved in the numerical simulations. In order to test the Virial theorem, Ballesteros-Paredes & Vázquez-Semadeni (1997, see also Ballesteros-Paredes et al. 1999a; Ballesteros Paredes 1999) calculated all the integrals in the EVT for clouds in numerical simulations of the interstellar medium at a kiloparsec scale by Passot et al. (1995). They found that the second time derivative of the moment of inertia never goes to zero, suggesting that clouds must be transient entities.

McKee (1999) has recognized the difficulty to achieving VE for actual MCs. He proposed two possibilities for assuming clouds in VE: the first one is to average in time the second time derivative of the moment of inertia. He suggests that $\langle \ddot{I} \rangle = 0$ if the averaging time considered is much larger than the dynamical timescale of the cloud, i.e., if $t_{\rm avg} \gg t_{\rm dyn}$. The second one is that for an ensemble of clouds, some of them may have positive values of $\ddot{I}$, and some others will have negative values, so that VE holds for the ensemble. Regarding the first assumption, there is a hidden assumption behind it: the cloud maintains its identity during several dynamical timescales[3]. However, from the numerical point of view, simulations of the ISM suggest that the clouds are continually morphing and exchanging mass, energy and momentum with their surroundings, so that over $t \sim t_{\rm dyn}$ they have changed significantly. On the other hand, observational evidence (Ballesteros-Paredes et al. 1999; Elmegreen 2000; Hartmann et al. 2001; Ballesteros-Paredes & Hartmann 2006) also suggests that the lifetimes of the clouds are not significantly larger than their own dynamical times (i.e., they are transient). Concerning the second assumption, the analysis by Ballesteros-Paredes & Vázquez-Semadeni (1997, see also Ballesteros Paredes 1999) shows that the moment of inertia spans up to 7 orders of magnitude (in absolute value) between the largest clouds and the smallest. Thus, it is not clear that an average for such large scatter will be representative of the actual dynamics of the clouds. In other words, even if $\ddot{I}$ did average out to zero for the ensemble, this does not alter the fact that, individually, clouds are not in VE.

[3] In fact, it has to be oscillating around a mean shape without a strong redistribution of mass, in order to achieve the condition $\langle \ddot{I} \rangle = 0$. A similar assumption is made by McKee and Zweibel (1992).

3.6 Sixth assumption: Larson's relationships are the observational demonstration of clouds being in Virial Equilibrium

The mean density-size and velocity dispersion-size relations first discussed by Larson (1981) are thought to be an observational demonstration of Virial Equilibrium. However, as Myers & Goodman (1988) recognize, they are actually only compatible with equipartition. For instance, assuming equipartition between kinetic (turbulent) and gravitational energy,

$$\delta v^2 \sim \frac{GM}{R} \propto \langle\rho\rangle R^2. \tag{18}$$

Thus, if there is a mean density-size power-law relationship $\rho \propto R^\alpha$, then, there should be a velocity dispersion-size relationship of the form

$$\delta v^2 \propto R^\beta, \tag{19}$$

with

$$\beta = (\alpha + 2)/2. \tag{20}$$

Several caveats must be mentioned at this point. First of all, this derivation does not mean VE, but energy equipartition, since the only assumption was $E_g \sim E_{\rm kin}$ (see eq. [18]). A similar result can be found if the equipartition assumed is valid between the gravitational and magnetic energy. In this case, δv is proportional to the Alfvén speed (see, e.g., Myers & Goodman 1988). Second, as discussed by Vazquez-Semadeni & Gazol (1995), in the particular case of $\rho \propto R^{-1}$, $\delta v \propto R^{1/2}$. However, the pair $\alpha = -1$, $\beta = 1/2$ is not unique. Any pair of values satisfying the equation (20) will be consistent with energy equipartition (again, not VE). Finally, it is convenient to recall that the validity of Larson's relations has been called into question, especially the mean density-size relationship (Kegel 1989; Scalo 1990; Vazquez-Semadeni et al. 1997; Ballesteros-Paredes & Mac Low 2002). It seems that this relationship is more a consequence of the observational process, in which the dynamical range of the observations is limited below by the minimum sensitivity of the telescopes, and above by saturation of the detectors, optically thick effects, and depletion.

4 DISCUSSION: VIRIAL MASS, VIRIAL EQUILIBRIUM AND THE STAR FORMATION EFFICIENCY

As it is discussed, there are at least six common assumptions related to the Virial Theorem which seem to be unjustified for a turbulent ISM. Some questions, such as why Virial

Masses are good order-of-magnitude estimations of the actual mass, or why in principle, the sub- or super-criticality of a molecular cloud core is a good estimation of the dynamical state of such a core, are still valid to ask.

The answer is that those estimations are based on energy equipartition, not on VE (i.e., $E_{\rm kin} \sim E_g \sim E_M$ does not mean that $\ddot{I}_L$ is negligible when it is compared to the left-hand side of eq. [2]). Molecular clouds seem to be in approximated equipartition between self-gravity, kinetic, and magnetic energy (see, e.g., Myers & Goodman 1988) although the super-Alfvénic nature of molecular clouds is still a matter of debate, (e.g., Padoan & Nordlund 1999, see also Ward-Thompson et al. 2006 for an observational review; Ballesteros-Paredes et al. 2006, for a theoretical one).

The question thus, is why MCs seem to be in energy equipartition? The answer is related to what we identify as a cloud, and to observational limitations: in the first case, clouds with a substantial excess of internal energy will rapidly expand and merge with the more diffuse medium. This is the case of an H II region or SN explosion expanding within its parental molecular cloud. The amount of energy provided by those events is enough to disrupt their parental environment within a few Myr (e.g., Franco et al. 1994). The excess of internal energy is mostly in the form of UV photons, which rapidly ionize the molecular gas, reducing even more the life timescale of what it is identified as the parental molecular cloud (see, e.g., Mellema et al. 2005). In the second case, if the cloud has an excess of gravitational energy, its free-fall time velocity is at most a factor of $\sqrt{2}$ the velocity dispersion needed for equipartition. This means that the difference between a free-fall collapsing cloud and one in equilibrium is just a factor of two in the energy, and a factor of $\sqrt{2}$ in velocity. Both systems will be, in principle, in order-of-magnitude energy equipartition. Note, furthermore, that in the free-fall collapsing case all the kinetic energy will be observationally identified as energy for support, where it is not actually providing any support. This re-enforces the need to distinguish between the compressible, expansive and vortical modes of the kinetic energy (see §3.1).

As discussed above, not all the terms entering the VT can be measured observationally. However, all of them can be measured from numerical simulations. The work by Ballesteros-Paredes & Vázquez-Semadeni (1995); Ballesteros-Paredes & Vázquez-Semadeni (1997); Shadmehri et al. (2002); Tilley & Pudritz (2004) for turbulent realizations of MCs shows that all the terms entering the VT are of similar importance. Note that the same problems related to the identification of an expanding cloud are applicable to the numerical work. In this case, what we identify as a cloud are the density enhancements. By definition, vacuumed regions (due either to modeled stellar activity, or by the turbulence itself) are no longer considered clouds. As for the collapsing case, strongly self-gravitating clouds will develop a velocity field that is a factor of $\sqrt{2}$ the value of energy equipartition, if only gravity is acting, and of the order of magnitude if the turbulence is forced. Thus, order of magnitude equipartition between magnetic, kinetic, and gravitational energy is valid to assume to predict whether the clouds should be supercritical or subcritical (Bertoldi & McKee 1992; Nakano 1998), or to understand why subcritical cores are unlikely to survive long-lifetimes within MCs (Nakano 1998). However, the order-of-magnitude coincidence of the involved energies does not mean that clouds are in equilibrium at all. It should be stressed that the difference is not just semantic (Virial mass vs energy-equipartition mass). The difference is conceptual: turbulent clouds cannot be in equilibrium because turbulent fragmentation takes them out of equilibrium, and thus they do not last long. In order to achieve equilibrium, (even if it is a time-averaged equilibrium, $\langle \ddot{I} \rangle = 0$, as suggested by McKee (1999)), it is necessary to allow the cloud to live for, at least, several crossing times. To give an example, the Taurus Molecular Cloud has a size of $l \sim 20$ pc, and a velocity dispersion of $\delta v \sim 2$ km sec^{-1}. Its lateral crossing time is $\tau_{\rm cross} = l/\delta v \sim 10$ Myr. In order to achieve equilibrium, Taurus has to live 20-30 Myr without a serious distortion or modification. Such a condition seems difficult to achieve if one realizes that Taurus has a Mach number of the order of 10, and the associated H I gas has Mach numbers of 20-30 in respect to the internal CO gas (see, e.g., the velocity-position diagrams of H I and CO in Ballesteros-Paredes et al. 1999).

The fact that turbulent clouds cannot be in VE and cannot last several crossing times is in clear contradiction with Tan et al. (2006), who made theoretical arguments that favor star formation occurring during several dynamical timescales. However, in their theoretical derivation they applied the star formation rate per free-fall time expression given by eq. (30) of Krumholz & McKee (2005) to a clump in hydrostatic equilibrium. As Krumholz & McKee (2005) point out, their eq. (30) is valid only for large Mach numbers[4] (20-40), while by definition, a clump in hydrostatic equilibrium is subsonic. Thus, the theoretical arguments in favor of slow star formation are wrong

Another point to be stressed concerns line profiles of H I, CO and its isotopes, and even higher density tracers (CS, NH_3, etc.). If interstellar clouds and/or their cores are not in Virial Equilibrium, they must be distorted and disrupted within a dynamical timescale. How will their line-profiles look, and how will they look if they were in equilibrium? Certainly, if turbulence were microscopic (necessary to achieve equilibrium), line profiles should be symmetric. The fact that H I and CO clouds (e.g., Hartmann & Burton 1997; Dame et al. 2001) as well as molecular cloud cores (e.g., Falgarone et al. 1998) exhibit non-symmetric line profiles is probably the best and the only evidence that the term $\ddot{I}_L$ is different from zero[5]. Even a small degree of asymme-

[4] Other problems involved in the derivation of eq (30) by Krumholz & McKee (2005) will be discussed in a further contribution.

[5] Note that in an Eulerian frame of reference, an idealized dust-lane in a spiral wave may appear to be long-lived in terms of its overall structure even though particular gas molecules do not stay inside it very long. For such system the line profile will be asymmetric, while its Eulerian second-time derivative of the moment of inertia can be zero ($\ddot{I}_E = 0$). This system, however, is not in VE, as the definition of VE, eq. (17) involves the *Lagrangian* frame of reference (Note that the difference between the Eulerian

try in the line profiles suggests that large-scale motions are present in the observed system, which is a natural consequence of turbulence being a multiscale phenomenon. This does not mean that such a system is collapsing or expanding as a whole, as formerly suggested by Goldreich & Kwan (1974). Instead, it means that turbulent large-scale motions are present in the system, which should be evolving within a dynamical timescale.

In this context, it should be stressed that clouds presenting large-scale motions and evolving to form stars rapidly not necessarily will have a high star formation efficiency, as is the common belief since Zuckerman & Evans (1974). Although former models of quasi-static evolution of molecular clouds were proposed to reduce the star formation efficiency, the turbulent models producing local collapse rapidly, have small efficiency because gravo-turbulent fragmentation involves only a small fraction of the mass of the system in collapsing regions (Vázquez-Semadeni et al. 2003, 2005). In numerical models, when energy feedback from stars is included, the cloud is blowed out rapidly (see, e.g., Ballesteros-Paredes 2004a). However, it should be recognized that a detailed quantification of the star formation efficiency in turbulent simulations with open boundary conditions and stellar feedback is needed.

5 CONCLUSIONS

The present contribution has discussed the applicability of the six more common assumptions on the Virial Theorem. Specifically,

(i) It was shown that a decomposition of the velocity field into its vortical and compressible modes is necessary, since only modes satisfying the condition $\nabla \cdot u > 0$ provide support, while modes satisfying $\nabla \cdot u < 0$ foment collapse.

(ii) It was argued that for a supersonic, turbulent ISM, surface terms should not be neglected.

(iii) It was shown that the gravitational term can be decomposed into a contribution from the cloud itself, and a contribution from the outside. The first part is the well-known gravitational energy. The second part is the sum of three terms: The gravitational pressure evaluated at the surface of the cloud plus three times the work done against the external mass to assemble the density distribution of the cloud, plus a term that depends on the gradient of the density distribution of the cloud. These represent the tidal forces due to an external potential, as could be the case of a dense core within a giant molecular cloud, or a giant molecular cloud close to a spiral arm. It is argued that this contribution can be as important as the self-gravitational energy.

(iv) Using a simple counter-example, it was shown that the sign of the second-time derivative of the moment of inertia does not determine whether the cloud is contracting or expanding. An expanding cloud may very well satisfy the condition $\ddot{I} < 0$, and a contracting one may satisfy $\ddot{I} > 0$, contrary to the common belief. In other words, $\ddot{I}$ has been treated in the literature as if it were $\dot{I}$.

(v) It was argued that interstellar clouds are not likely to satisfy the Virial Equilibrium (VE) condition $\ddot{I} = 0$.

(vi) It was shown that Larson's (1981) relations are not observational proof for clouds being in VE.

(vii) Clouds seem to be in energy equipartition because of either observational limitations, as well as because of the intrinsic definition of a cloud.

Turbulent fragmentation plays a crucial role for the inapplicability of the VT to interstellar clouds, since it will induce a flux of mass, moment and energy between the clouds and their environment, and will favor local collapse while disrupting the clouds within a dynamical timescale. The common assumptions discussed in the present contribution drive our understanding of the dynamical state of the interstellar clouds toward a picture that favors a static ISM. However, they are highly difficult to fulfill if the ISM is highly turbulent, as it was found to be many years ago (e.g., McCray & Snow 1979). Inferences of the star formation efficiency for supersonic (Mach numbers $\sim 20 - 40$) clouds in virial equilibrium living several dynamical timescales (e.g., Krumholz & McKee 2005; Tan et al. 2006) should be taken with caution.

The lack of observational evidence for clouds being in VE, and the identificaion of asymmetrical line profiles observed toward interstellar clouds using different tracers are the best evidence of clouds being out-of-equilibrium systems. These facts lead us to the conclusion that clouds should be transient structures, which exchange mass, momentum and energy with their environment. This is precisely the opposite point of view to the old one in which clouds should be at rest, which still is present in textbooks and papers, but it is consistent with recent observationally-based works that favor a scenario of rapid cloud and star formation (see, e.g., the reviews by Mac Low and Klessen 2004; Ballesteros-Paredes et al. 2006, and references therein.).

and Lagrangian VT is $\ddot{I}_L - \ddot{I}_E = -4\tau_{\rm kin} + d\Phi/dt$, as pointed out by McKee and Zweibel 1992).

ACKNOWLEDGEMENTS

I want to thank Paola D'Alessio, and Hebe Vessuri who encouraged me to write this paper. To Paola D'Alessio, Lee Hartmann, Ralf Klessen, Stan Kurtz, Mordecai-M. Mac Low, Padelis Papadopoulos and Enrique Vázquez-Semadeni, for the careful reading of this manuscript and their useful comments. Special thanks to Lee and Enrique for fruitful and enjoyable discussions on the subject, and to Bruce Elmegreen, the referee, for an encouraging referee report. Thanks also to Mary-Ann Hall (mahall@mail.unla.edu.mx) for proofreading this paper. This work was supported by UNAM-PAPIIT grant number 110606, and made extensive use of the NASA-ADS database.

REFERENCES

Ballesteros Paredes, J. 1999, Ph.D. Thesis,
Ballesteros-Paredes, J. 2004, Ap& SS, 289, 243

Ballesteros-Paredes, J. 2004, Ap& SS, 292, 193
Ballesteros-Paredes, J., & Hartmann, L. 2006. Rev. Mex. Astron. Astrophys, submitted (`astro-ph/0605268`)
Ballesteros-Paredes, J., Hartmann, L., & Vázquez-Semadeni, E. 1999, ApJ, 527, 285
Ballesteros-Paredes, J., Klessen, R. S., Mac Low, M. -., & Vazquez-Semadeni, E. 2006, in Protostars and Planets V. (`astro-ph/0603357`)
Ballesteros-Paredes, J., & Mac Low, M.-M. 2002, ApJ, 570, 734
Ballesteros-Paredes, J., & Vázquez-Semadeni, E. 1995, Revista Mexicana de Astronomía y Astrofísica, Conf. Series, 3, 105
Ballesteros-Paredes, J., & Vázquez-Semadeni, E. 1997, American Institute of Physics Conference Series, 393, 81
Ballesteros-Paredes, J., Vázquez-Semadeni, E., and Scalo, J. 1999a, ApJ, 515, 286–303
Bertoldi, F., & McKee, C. F. 1992, ApJ, 395, 140
Bonazzola, S., Heyvaerts, J., Falgarone, E., Perault, M., & Puget, J. L. 1987, A& A, 172, 293
Boulares, A., & Cox, D. P. 1990, ApJ, 365, 544
Bowyer, S., Lieu, R., Sidher, S. D., Lampton, M., & Knude, J. 1995, Nature, 375, 212
Carr, J. S. 1987, ApJ, 323, 170
Chandrasekhar, S. (1951) Proc. R. Soc. London A, 210, 26
Chandrasekhar, S., & Fermi, E. 1953, ApJ, 118, 116
Dame, T. M., Hartmann, D., & Thaddeus, P. 2001, ApJ, 547, 792
Dib, S., et al. 2006, in preparation
Elmegreen, B. G. 2000, ApJ, 530, 277
Elmegreen, B. G., & Scalo, J. 2004, ARA&A, 42, 211
Falgarone, E., Panis, J.-F., Heithausen, A., Perault, M., Stutzki, J., Puget, J.-L., & Bensch, F. 1998, A& A, 331, 669
Falgarone, E., Phillips, T. G., & Walker, C. K. 1991, ApJ, 378, 186
Franco, J., Shore, S. N., & Tenorio-Tagle, G. 1994, ApJ, 436, 795
Fuller, G. A., & Myers, P. C. 1992, ApJ, 384, 523
Gazol, A., Vázquez-Semadeni, E., & Kim, J. 2005, ApJ, 630, 911
Goldreich, P., & Kwan, J. 1974, ApJ, 189, 441
Hartmann, L. 1998, Accretion processes in star formation. Cambridge University Press (Cambridge:New York)
Hartmann, L., Ballesteros-Paredes, J., & Bergin, E. A. 2001, ApJ, 562, 852
Hartmann, D., & Burton, W. B. 1997, Atlas of Galactic Neutral Hydrogen. Cambridge University Press
Heitsch, F., Mac Low, M.-M., and Klessen, R. S. 2001, ApJ, 547, 280–291
Heyer, M. H., Carpenter, J. M., & Snell, R. L. 2001, ApJ, 551, 852
Hunter, J. H., Jr., & Fleck, R. C., Jr. 1982, ApJ, 256, 505
Jenkins, E. B. 2002, ApJ, 580, 938
Jenkins, E. B., Jura, M., & Loewenstein, M. 1983, ApJ, 270, 88
Jenkins, E. B., & Tripp, T. M. 2001, ApJS, 137, 297
Jijina, J., Myers, P. C., & Adams, F. C. 1999, ApJS, 125, 161
Kegel, W. H. 1989, A& A, 225, 517
Keto, E. R., & Myers, P. C. 1986, ApJ, 304, 466
Klessen, R. S., Heitsch, F., and Mac Low, M.-M. (2000) ApJ, 535, 887–906
Kolmogorov, A. N. 1941, Dokl. Akad. Nauk SSSR, 30, 301–305
Kornreich, P., & Scalo, J. 2000, ApJ, 531, 366
Krumholz, M. R., & McKee, C. F. 2005, ApJ, 630, 250
Larson, R. B. 1981, MNRAS194, 809–826
Lesieur, M. 1990. Turbulence in Fluids (Dordrecht:Kluwer)
Lequeux, J. 2005, The interstellar medium. 2005 EDP Sciences, Astronomy and astrophysics library, (Berlin: Springer)
Loren, R. B. 1989, ApJ, 338, 925
Mac Low, M.-M., Balsara, D. S., Kim, J., & de Avillez, M. A. 2005, ApJ, 626, 864
Mac Low, M.-M. and Klessen, R. S. 2004, Rev. Mod. Phys., 76, 125–194
McCray, R., & Snow, T. P., Jr. 1979, ARA&A, 17, 213
McCrea, W. H. 1957, MNRAS, 117, 562
McKee, C. F. 1999, NATO ASIC Proc. 540: The Origin of Stars and Planetary Systems, 29
McKee, C. F. and Zweibel, E. G. 1992, ApJ399, 551–562
Mellema, G., Arthur, S. J., Henney, W. J., Iliev, I. T., & Shapiro, P. R. 2005, ArXiv Astrophysics e-prints, arXiv:astro-ph/0512554
Moriarty-Schieven, G. H., Andersson, B.-G., & Wannier, P. G. 1997, ApJ, 475, 642
Myers, P. C., & Goodman, A. A. 1988, ApJL, 326, L27
Nakano, T. 1998, ApJ, 494, 587
Padoan, P. 1995, MNRAS277, 377–388
Padoan, P., & Nordlund, Å. 1999, ApJ, 526, 279
Parker, E. N. 1969, Space Science Reviews, 9, 651
Parker, E. N. 1979. Cosmical magnetic fields: Their origin and their activity. Oxford University Press (Oxford, Clarendon)
Passot, T., Vazquez-Semadeni, E., & Pouquet, A. 1995, ApJ, 455, 536
Redfield, S., & Linsky, J. L. 2004, ApJ, 613, 1004
Sasao, T. 1973, Publ. Astron. Soc. Jap., 25, 1–33
Scalo, J. M. 1988, LNP Vol. 315: Molecular Clouds, MilkY-Way and External Galaxies, 315, 201
Scalo, J. 1990, ASSL Vol. 162: Physical Processes in Fragmentation and Star Formation, 151
Shadmehri, M., Vázquez-Semadeni, E., & Ballesteros-Paredes, J. 2002, ASP Conf. Ser. 276: Seeing Through the Dust: The Detection of HI and the Exploration of the ISM in Galaxies, 276, 190
Shu, F. 1991. The Physics of the Astrophysics. II. Gas dynamics. Published by University Science Books, (New York)
Spitzer, L. 1978. Physical processes in the Interstellar Medium. New York Wiley-Interscience
Stahler, S. W., & Palla, F. 2005, The Formation of Stars, by Steven W. Stahler, Francesco Palla, Wiley-VCH
Tan, J. C., Krumholz, M. R., & McKee, C. F. 2006, ApJL, 641, L121
Tilley, D. A., & Pudritz, R. E. 2004, MNRAS, 353, 769
Vazquez-Semadeni, E., Ballesteros-Paredes, J., & Ro-

driguez, L. F. 1997, ApJ, 474, 292
Vázquez-Semadeni, E., Ballesteros-Paredes, J., & Klessen, R. S. 2003, ApJL, 585, L131
Vazquez-Semadeni, E., & Gazol, A. 1995, A& A, 303, 204
Vázquez-Semadeni, E., Gazol, A., Passot, T., & Sánchez-Salcedo, J. 2003, LNP Vol. 614: Turbulence and Magnetic Fields in Astrophysics, 614, 213
Vázquez-Semadeni, E., Kim, J., & Ballesteros-Paredes, J. 2005, ApJL, 630, L49
Vázquez-Semadeni, E., Passot, T., & Pouquet, A. 1996, ApJ, 473, 881
von Weizsäcker, C. F. 1951, ApJ, 114, 165–186.
Ward-Thompson, D., Andre, P., Crutcher, R., Johnstone, D., Onishi, T., & Wilson, C. 2006, (astro-ph/0603474)
Williams, J. P., & Maddalena, R. J. 1996, ApJ, 464, 247
Yonekura, Y., Dobashi, K., Mizuno, A., Ogawa, H., & Fukui, Y. 1997, ApJS, 110, 21
Zuckerman, B., & Evans, N. J., II 1974, ApJL, 192, L149